Longtime behavior of stochastic processes

Workshop Booklet

June 2–6, 2025

Clermont-Ferrand, France

Organized by Manon Michel (`manon.michel@uca.fr`)

Welcome and Practical Information

Workshop Venue

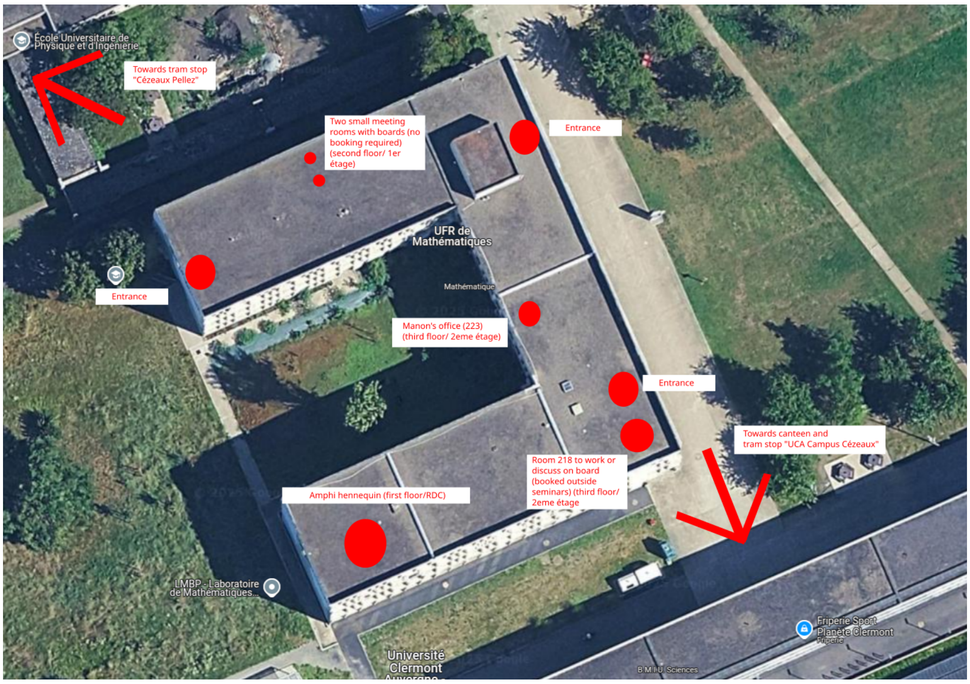
The workshop will take place at:

LMBP (Laboratoire de Mathématiques Blaise Pascal)
Université Clermont Auvergne (UCA)
Amphi Hennequin
Campus des Cézeaux
3, place Vasarely 63178 Aubière Cedex, France
`https://lmbp.uca.fr/`

Participants will be officially welcomed on Monday at **13:40** with coffee. The first talk starts at **14:00**.

If you arrive earlier, feel free to come by the lab in the morning — space will be available for discussions and informal work:

- **Amphi Hennequin** (first floor): main conference room.
- **Room 218** (third floor): dedicated space for workshop participants, equipped with a large blackboard. This is your go-to spot for informal discussions or work. Please note that if a seminar is scheduled, this room may temporarily be unavailable.
- **Room 223** (third floor): office of **Manon**, located right next to Room 218.
- **Meeting Rooms** (second floor): these rooms are freely available, equipped with boards and can accommodate up to 4 persons.



Accommodation

Your accommodation is booked at:

Maison Internationale Universitaire (MIU)
9 Rue Kessler, 63000 Clermont-Ferrand, France
`http://www.miu-clermont.fr/Modeles/page_presentation.php?&lang=en`

Transportation

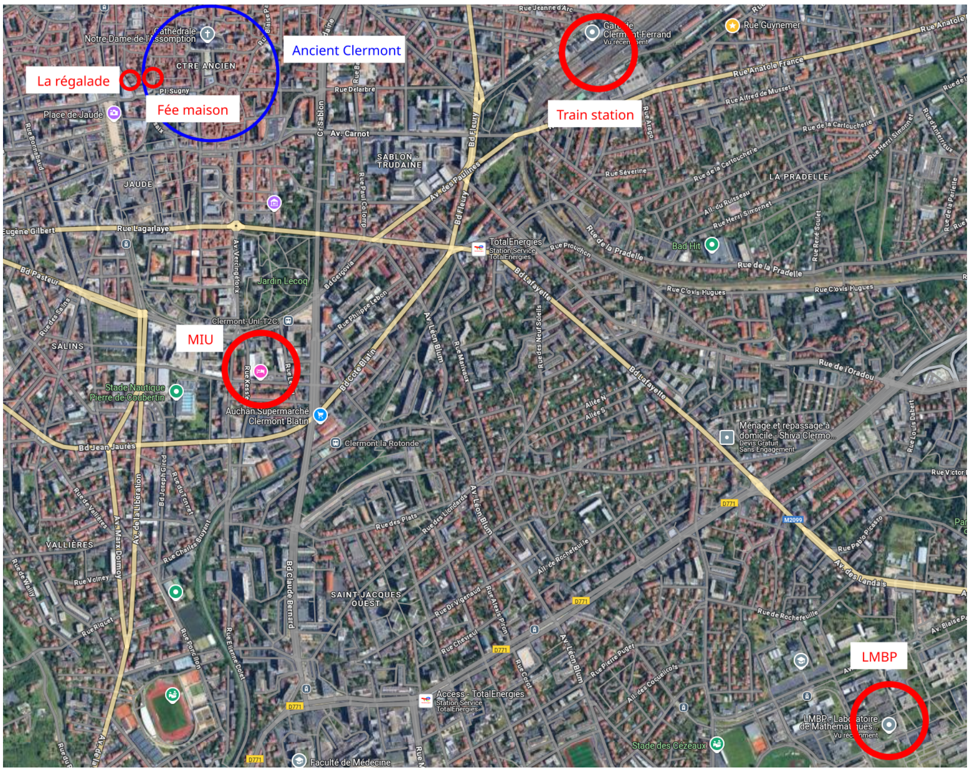
From Clermont-Ferrand train station:

- 20-minute walk to MIU or take Bus B (Gare SNCF → UCA - Campus Centre, direction: Royat Pl. Allard)
- 30-minute walk to LMBP or take Tram (Delille Salford → Cézeaux-Pellez) or Bus 13 (Esplanade → Observatoire, direction: Quartier Chambon)

From MIU to LMBP: Direct tram from UCA - Campus Centre to Cézeaux-Pellez (20 min, direction: Les Vergnes). Walking takes 30–40 minutes.

Transport Resources:

- `https://www.t2c.fr` (Transportation website)
- `https://www.t2c.fr/sites/default/files/Nouvelle_arborescence/T2C-Plan%20TRAVAUX%202023-2024-ETE-SANS-CARROY-775X700-WEB%20suppression%20C%20la%20navette.pdf` (Tram/Bus map)



Meals and Dinners

- **Monday Dinner:** Fée Maison – 32 rue des petits gras, 63000 Clermont-Ferrand
- **Wednesday Dinner:** La Régalade – 9 rue Nestor Perret, 63000 Clermont-Ferrand
- **Lunches:** Caféteria Le Saxo on campus (coupons will be made available)
- **Friday Lunch:** Will be held earlier at **11:30 AM** to accommodate those taking the 13:28 train.

Activities

- **Monday:** After the talks, we'll take a guided stroll around Clermont's historical center, starting at 17:45 from Maison du Tourisme, place de la Victoire. This will be followed by dinner. There will be a group departure from the lab.
- **Wednesday:** Hike to the Puy de Dôme! It's an easy trail, but bring suitable shoes, rain gear, and sun protection. The bus departs at 2:20 PM from the parking next to the tram stop Cézeaux-Pellez. We'll head together to dinner once back to Clermont.

Online Info

All updated information and schedule will be available on the ANR SUSA website:

`https://anr-susa.math.cnrs.fr/page/events/`

Program

Day 1 – June 2, 2025

09:00 *Free Discussion at LMBP*

13:40 **Welcome and Coffee**

14:00 **Pierre Lebris** (IHES)
Uniform in time mean field limit in the metastable Curie-Weiss model

14:40 **Thibaut Arnoulx de Pirey** (CEA Saclay)
Dynamical slowdown in high-dimensional population dynamics

15:40 Coffee Break

16:10 **Grégory Schehr** (Sorbonne Université)
Large Deviations in Switching Diffusion: from Free Cumulants to Dynamical Transitions

17:45 Ancient Clermont Tour

20:00 Dinner *La Fée Maison*

Day 2 – June 3, 2025

09:00 *Free Discussion at LMBP*

09:20 **Vivien Lecomte** (Université Grenoble Alpes)
Generalized Kirchoff's Current Law(s)

10:20 **Sophie Hermann** (Sorbonne Université)
Noether's theorem and hyperforces in statistical mechanics

11:00 Coffee Break

11:30 **Andreas Eberle** (Universität Bonn)
Non-reversible lifts of reversible diffusion processes and relaxation times

12:30 Lunch

14:00 **Richard Blythe** (University of Edinburgh)
Cluster dynamics of dense interacting run-and-tumble particles in 1d

15:00 **Leo Hahn** (Université de Neuchâtel)
Investigating the Shape Transition of Run-and-Tumble Particles: A One-Dimensional PDMP Approach to the Regularity of the Invariant Measure

15:40 Coffee Break

16:10 **Max Fathi** (Sorbonne Université)
The cutoff phenomenon: old and new

Day 3 – June 4, 2025

09:00 *Free Discussion at LMBP*

09:20 **Lucas Journel** (Université Paris-Saclay)
Uniform convergence of the Fleming-Viot process for the sampling of quasi-stationary measures

10:20 **Camille Aron** (ENS Paris, EPFL)
Slowing the butterflies: From stochastic resets in nonlinear maps to quenched disorder in metals

11:00 Coffee Break

11:30 **Tristan Benoist** (Université Toulouse)
Long time behavior of quantum trajectories

12:30 Lunch

14:20 Departure for Puy de Dôme Hike

19:45 Dinner *La Régalade*

Day 4 – June 5, 2025

09:00 *Free Discussion at LMBP*

09:20 **Julien Reygner** (École des Ponts ParisTech)
Local convergence rates for Wasserstein gradient flows and McKean-Vlasov equations with multiple stationary solutions

10:20 **Benoît Dagallier** (Université Paris-Dauphine)
Log-Sobolev inequalities for mean-field particle systems

11:00 Coffee Break

11:30 **Aurélien Grabsch** (Sorbonne Université)
Tracer and current fluctuations in 1D diffusive systems

12:30 Lunch

14:00 **Francis Lörler** (Universität Bonn)
Convergence of non-reversible lifts via flow Poincaré inequality with application to self-repelling motion

15:00 **Tony Jin** (Université Côte d'Azur)
An exact representation of the dynamics of quantum spin chains as classical stochastic processes with particle/antiparticle pairs

15:40 Coffee Break

16:10 **Kirone Mallick** (CEA Saclay)
Some exact results on Quantum Walks

Day 5 – June 6, 2025

09:00 **Pierre Illien** (Sorbonne Université)
Non-Gaussian density fluctuations in the Dean-Kawasaki equation

10:00 Coffee Break

10:30 **Christophe Bahadoran** (Université Clermont Auvergne)
Invariant measures, hydrodynamics and relaxation limits for multilane exclusion processes

11:30 Lunch

Abstracts

Thibaut Arnoulx de Pirey (CEA Saclay)
Dynamical slowdown in high-dimensional population dynamics
I will discuss recent results on the long-time behavior of high-dimensional deterministic models of population dynamics. Using an exactly solvable model, I will show that dynamical slowdown generically arises due to the existence of many absorbing states (namely, if a population goes extinct, it remains extinct forever). In parallel, at long times, the steady-state measure is found to concentrate around unstable fixed points of the deterministic dynamics, the properties of which simple counting arguments on the ensemble of possible fixed points fail to account for. I will conclude the presentation by extending these results to more complex canonical models of population dynamics and by discussing possible connections to the dynamics of real-world high-diversity ecosystems.

Camille Aron (ENS Paris, EPFL)
Slowing the butterflies: From stochastic resets in nonlinear maps to quenched disorder in metals
The spatiotemporal scrambling of information in chaotic many-body systems not only underpins the foundations of statistical physics but also presents significant challenges for the reliable operation of (quantum) information processing devices as system complexity increases. We demonstrate how key diagnostics of chaos—specifically, the Lyapunov exponent and the butterfly velocity—can be systematically suppressed by stochastically resetting dynamical systems to their initial conditions. Analogous to how the Navier–Stokes equations provide an effective description of fluid dynamics, we discuss the potential formulation of an effective field theory for information scrambling, capable of capturing the coarse-grained dynamics of information spreading in space and time. This perspective connects minimal nonlinear models, such as the logistic map, with more complex instances of interacting quantum systems, such as interacting diffusive metals.

Christophe Bahadoran (Université Clermont Auvergne)
Invariant measures, hydrodynamics and relaxation limits for multilane exclusion processes
The totally asymmetric exclusion process (TASEP) is a fundamental stochastic model in nonequilibrium statistical physics, where particles with mutual exclusion hop on the doubly infinite 1d lattice subject to an external field . It is related to random polymers, queuing networks and growth models, as well as a simplified traffic-flow model. I will describe recent results on multilane exclusion processes, where exclusion processes on different lanes interact through lane changes:
- The structure of invariant measures, which interpolates between known 1d results and conjectural multi-d results.
- The hydrodynamic limit, i.e. the evolution of the macroscopic density field, with phase transitions related to lane competition.
- Relaxation limits for the microscopic dynamics and related PDEs in the fast lane-change limit.
(Joint works with G. Amir, O. Busani and E. Saada)
References:
Invariant measures for multilane exclusion processe. To appear in Ann. Inst. Poincaré Probabilités et Statistique. ArXiv: 2105.12974
Hydrodynamics and relaxation limits for multilane exclusion processes and related hyperbolic systems. ArXiv: 2501.19355

Tristan Benoist (Université Toulouse)
Long time behavior of quantum trajectories
Quantum trajectories are Markov processes modeling the evolution of a system subjected to repeated (indirect) measurement. The prototypical example of an experience they describe is Haroche's group experiment measuring the number of photons in a superconducting cavity

using Rydberg atoms. They are used on a daily basis in quantum optics. In this presentation I will review recent results on the longtime behavior of quantum trajectories. In particular I will detail the notion of “ purification “ central to these processes. I will explain how it is used to prove the uniqueness of the invariant measure, spectral gap and several related limit theorems.

Richard Blythe (University of Edinburgh)
Cluster dynamics of dense interacting run-and-tumble particles in 1d
(with Peter Sollich, Martin Evans and Satya Majumdar)
Run-and-tumble particles (known also as persistent random walkers) exhibit a motion inspired by certain organisms (such as bacteria and fish), whereby particles seek to maintain a predetermined speed in some direction, but change direction as a stochastic process. When such particles interact, for example through hard-core exclusion, detailed balance is broken and the statistical properties of dense systems defy a detailed understanding. In this talk I will summarise progress with the one-dimensional problem, focussing particularly on the dynamics of clusters that form in the limit where directional changes are rare. We show that these dynamics can be expressed as a coagulation-fragmentation process that in a continuum limit takes the form of an advection-diffusion equation. A particular challenge is that the advection coefficient is time-dependent. Despite this, we are able to gain some insights into the nature of the relaxation to stationarity, although a number of open problems remain.

Benoît Dagallier (Université Paris-Dauphine)
Log-Sobolev inequalities for mean-field particle systems.
We consider the (overdamped) Langevin dynamics associated with particles interacting through a smooth mean-field potential and attempt to quantify its speed of convergence as a function of the number N of particles and the temperature. The main goal is to find a criterion relating fast convergence speed, i.e. independent of N, to properties of the static free energy of the model.
We show that a certain notion of convexity of the free energy implies uniform-in-N bounds on the convergence speed, measured through a log-Sobolev inequality. In some cases this convexity criterion is sharp, for instance in the Curie-Weiss model where convexity holds up to the critical temperature.
Our proof does not involve the dynamics. Instead, we decompose the measure describing interactions between particles with inspiration from renormalisation group arguments for lattice models, that we adapt here to a lattice-free setting in the simplest case of mean-field interactions. Our results apply more generally to non mean-field, possibly random settings, provided each particle interacts with sufficiently many others.
Based on joint work with Roland Bauerschmidt and Thierry Bodineau.

Andreas Eberle (Universität Bonn)
Non-reversible lifts of reversible diffusion processes and relaxation times
We propose a new concept of lifts of reversible diffusion processes and show that various well-known non-reversible Markov processes arising in applications are lifts in this sense of simple reversible diffusions. Furthermore, we introduce a concept of non-asymptotic relaxation times and show that these can at most be reduced by a square root through lifting, generalising a related result in discrete time.
For reversible diffusions on domains in Euclidean space, or, more generally, on a Riemannian manifold with boundary, non-reversible lifts are in particular given by the Hamiltonian flow on the tangent bundle, interspersed with random velocity refreshments, or perturbed by Ornstein-Uhlenbeck noise, and reflected at the boundary. In order to prove that for certain choices of parameters, these lifts achieve the optimal square-root reduction up to a constant factor, precise upper bounds on relaxation times are required. We demonstrate how the recently developed approach to quantitative hypocoercivity based on space-time Poincaré inequalities can be rephrased and simplified in the language of lifts and how it can be applied to find optimal lifts.

This is joint work with Francis Lörler (Bonn).

Max Fathi (Sorbonne Université)
The cutoff phenomenon: old and new
In this talk, I will discuss cutoff for Markov chains, a phenomenon where the profile of convergence to equilibrium for a sequence of Markov chains becomes more and more abrupt. I will present the context, and some recent developments by Justin Salez and his collaborators, based on functional inequalities and curvature. If time allows, I will discuss some recent work with Djalil Chafaï and Nikita Simonov on cutoff for nonlinear PDE, and a few open problems.

Aurélien Grabsch (Sorbonne Université)
Tracer and current fluctuations in 1D diffusive systems
In this talk, I will review recent results obtained on the symmetric simple exclusion process (SEP) and other models of 1D diffusive systems. At large scales, these models can be studied within the framework of macroscopic fluctuation theory (MFT). For a few models, like the SEP, the MFT equations are classically integrable and can be solved to obtain exact results. However for most systems this is not the case, but several results can still be obtained in this case and applied to realistic models of interacting particles.

Leo Hahn (Université de Neuchâtel)
Investigating the Shape Transition of Run-and-Tumble Particles: A One-Dimensional PDMP Approach to the Regularity of the Invariant Measure
The invariant measure of a wide class of run-and-tumble particles (RTPs) subjected to a potential possesses a density, which may be either continuous or discontinuous, depending on model parameters. This key feature, known as shape transition, indicates whether the system is close to equilibrium (continuous density) or substantially departs from it (discontinuous density). Building on and extending existing results concerning the regularity of the invariant measure of one-dimensional piecewise-deterministic Markov processes (PDMPs), I will show how to characterize the shape transition even in situations where the invariant measure cannot be computed explicitly. This analysis confirms shape transition as a robust, general feature of RTPs under a potential, and also refines the regularity theory for the invariant measure of one-dimensional PDMPs.

Sophie Hermann (Sorbonne Université)
Noether's theorem and hyperforces in statistical mechanics
Noether's theorem is familiar to most physicists due its fundamental role in linking the existence of conservation laws to the underlying symmetries of a physical system. I will present how Noether's reasoning also applies within statistical mechanics to thermal systems, where fluctuations are paramount. Exact identities ("sum rules") follow thereby from functional symmetries. The obtained sum rules contain both, well-known relations, such as the first order term of the Yvon-Born-Green (YBG) hierarchy (i.e. the spatially resolved force balance), as well as previously unknown identities, relating different correlations in many-body systems. The identification of the underlying Noether concept enables their systematic derivation. Since Noether's theorem is quite general it is possible to generalize to arbitrary thermodynamic observables. This generalization yields sum rules for hyperforces, i.e. the mean product between the considered observable and the relevant forces that act in the system. Simulations of a range of simple and complex liquids demonstrate the fundamental role of these correlation functions in the characterization of spatial structure, such as quantifying spatially inhomogeneous self-organization. Finally, we show that the considered phase-space-shifting is a gauge transformation in equilibrium statistical mechanics.

Pierre Illien (Sorbonne Université)
Non-Gaussian density fluctuations in the Dean-Kawasaki equation
Computing analytically the n-point density correlations in systems of interacting particles is

a long-standing problem of statistical physics, with a broad range of applications, from the interpretation of scattering experiments in simple liquids, to the understanding of their collective dynamics. For Brownian particles, i.e. with overdamped Langevin dynamics, the microscopic density obeys a stochastic evolution equation, known as the Dean-Kawasaki equation. In spite of the importance of this equation, its complexity makes it very difficult to analyze the statistics of the microscopic density beyond simple Gaussian approximations. In this work, resorting to a path-integral description of the stochastic dynamics and relying on the formalism of macroscopic fluctuation theory, we go beyond the usual linearization of the Dean-Kawasaki equation, and we compute perturbatively the three-point density correlation functions, in the limit of high-density and weak interactions between the particles. This exact result opens the way to using the Dean-Kawasaki beyond the simple Gaussian treatments, and could find applications to understand many fluctuation-related effects in soft and active matter systems.

Tony Jin (Université Côte d'Azur)
An exact representation of the dynamics of quantum spin chains as classical stochastic processes with particle/antiparticle pairs
Since the advent of quantum mechanics, classical probability interpretations have faced significant challenges. A notable issue arises with the emergence of negative probabilities when attempting to define the joint probability of non-commutative observables. In this talk, I will propose a resolution to this dilemma by introducing an exact representation of the dynamics of quantum spin chains using classical continuous-time Markov chains (CTMCs). These CTMCs effectively model the creation, annihilation, and propagation of pairs of classical particles and antiparticles. The quantum dynamics then emerges by averaging over various realizations of this classical process.

Lucas Journel (Université Paris-Saclay)
Uniform convergence of the Fleming-Viot process for the sampling of quasi-stationary measures
The Fleming-Viot process is a mean-field particle system whose empirical measure is an estimator of the law of a Markov process conditioned on its survival (in a way that we will define). In this talk I will present how to show the convergence of this estimator at a rate independent of time, and get exponential concentration inequalities in a soft killing case.
It is based on a work with Mathias Rousset from Inria Rennes.

Vivien Lecomte (Université Grenoble Alpes)
Generalized Kirchoff's Current Law(s)
In 1847, Kirchoff derived laws governing stationary currents and potential in electrical circuits, encoding conservation laws. They can be expressed in a local form (fluxes balance at nodes of the networks), or in a global form (where they express linear dependences between currents). Interestingly, such settings can be generalized to Markov chains (the conservation now being that of probability), and they imply that the stationary state can be expressed as a normalized sum of weights of covering trees of the graph between states (through "Kirchhoff's formula" a.k.a. Markov Chain Tree Theorem). I will show that, in such settings, a surprisingly simple relation comes as a consequence of these laws: a mutual linearity between currents (stating that any stationary current is an affine function of any other set of –independent enough– current). (Chemical) reactions networks are a generalization of such systems, where the network between species is now described by a hypergraph. I will explain how one can formulate a generalization of Kirchoff's Current Law applying to these, and describe the associated geometrical interpretation. Finally, an application to the problem of metabolic reconstruction in constrained-based models will be detailed.
Work based on collaborations with Sara Dal Cengio, Pedro Harunari, Matteo Polettini and Delphine Ropers.

Pierre Lebris (IHES)
Uniform in time mean field limit in the metastable Curie-Weiss model
Some low temperature particle systems in mean-field interaction are ergodic with respect to a unique invariant measure, while their (non-linear) mean-field limit may possess several steady states. In particular, in such cases, propagation of chaos (i.e. the convergence of the particle system to its mean-field limit as n, the number of particles, goes to infinity) cannot hold uniformly in time since the long-time behaviors of the two processes are a priori incompatible. However, the particle system may be metastable, and the time needed to exit the basin of attraction of one of the steady states of its limit, and go to another, is exponentially (in n) long. Before this exit time, the particle system reaches a (quasi-)stationary distribution, which we expect to be a good approximation of the corresponding non-linear steady state.
The goal of this talk is to study a toy model, the Curie-Weiss model, and show uniform in time propagation of chaos of the particle system conditioned to keeping a positive mean.
Based on joint work with L. Journel (Université de Neuchâtel).

Francis Lörler (Universität Bonn)
Convergence of non-reversible lifts via flow Poincaré inequality with application to self-repelling motion
Convergence to equilibrium of non-reversible Markov processes can be quantified via an extended Poincaré-type inequality in space and time, termed the flow Poincaré inequality. We demonstrate how such a flow Poincaré inequality can be obtained for a large class of processes using the framework of second-order lifts. The result can be applied to obtain quantitative convergence rates for an event chain algorithm for the harmonic chain, which is closely related to a discretisation of the self-repelling motion introduced by B. Tóth and W. Werner.
The talk is based on joint work with Andreas Eberle, Arnaud Guillin, Leo Hahn and Manon Michel.

Kirone Mallick (CEA Saclay)
Some exact results on Quantum Walks
Quantum analogs of classical random walks have been defined in quantum information theory as a useful concept to implement algorithms. Due to interference effects, statistical properties of quantum walks can drastically differ from their classical counterparts, leading to much faster computations.
Quantum walks define naturally a quantum dynamical system to which the counterparts of many questions on classical random walks can be generalized; they provide an ideal framework to study analytically the interplay between statistical and quantum effects. We shall present some exact results on continuous-time quantum walks on a lattice, such as survival properties of quantum particles in the presence of traps, growth of a quantum population in the presence of a source, and quantum return probabilities (Loschmidt echoes).

Julien Reygner (École des Ponts ParisTech)
Local convergence rates for Wasserstein gradient flows and McKean-Vlasov equations with multiple stationary solutions
Non-linear versions of log-Sobolev inequalities, that link a free energy to its dissipation along the corresponding Wasserstein gradient flow (i.e. corresponds to Polyak-Lojasiewicz inequalities in this context), are known to provide global exponential long-time convergence to the free energy minimizers, and have been shown to hold in various contexts. However they cannot hold when the free energy admits critical points which are not global minimizers, which is for instance the case of the granular media equation in a double-well potential with quadratic attractive interaction at low temperature. This work addresses such cases, extending the general arguments when a log-Sobolev inequality only holds locally and, as an example, establishing such local inequalities for the granular media equation with quadratic interaction either in the one-dimensional symmetric double-well case or in higher dimension in the low temperature regime.

The method provides quantitative convergence rates for initial conditions in a Wasserstein ball around the stationary solutions. The same analysis is carried out for the kinetic counterpart of the gradient flow, i.e. the corresponding Vlasov-Fokker-Planck equation. The local exponential convergence to stationary solutions for the mean-field equations, both elliptic and kinetic, is shown to induce for the corresponding particle systems a fast (i.e. uniform in the number or particles) decay of the particle system free energy toward the level of the non-linear limit. This is a joint work with Pierre Monmarché.

Grégory Schehr (Sorbonne Université)

Large Deviations in Switching Diffusion: from Free Cumulants to Dynamical Transitions

We study the diffusion of a particle whose diffusion constant changes randomly over time. Specifically, it switches between values drawn from a given distribution at a fixed rate. Using a renewal approach, we derive exact expressions for the moments of the particle's position at any finite time and for any distribution with well-defined moments. In the long-time limit, we show that the cumulants of the position grow linearly with time and are directly related to the free cumulants of the underlying distribution of diffusion constants. For specific cases, we analyze the large deviations of the particle's position, revealing rich behaviors and dynamical transitions in the rate function.